Diffusivelike buffering and saturation of large rivers

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We use mass balance data from Asian rivers together with a first-order diffusive simplification of the St. Venant-Exner equations to characterize river floodplain processes and discuss the reaction of a large model river to a hillslope supply of eroded masses. The simple analytical solution derived for the long-term profile of the river bed shows that (i) the system converges towards a state in which it reacts to perturbations in erosion of the landscape by small-amplitude oscillations around an average ''stationary'' state, (ii) to have an effective influence on the river plain profile, the perturbations need to have frequencies smaller than the characteristic frequencies of the river system, and (iii) this river buffering might be linked with a possible long-term saturation of the system carrying capacity. [S1063-651X(99)12711-9]

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I. INTRODUCTION

Most of the models that attempt to explain the general form of landscapes do it through slope processes whether cascading, like avalanches or landslides, or diffusive coupled to the development of a drainage network that carries the mass out of the eroding regions [1–7]. All these models are constrained by several statistical laws, such as the Horton-Strahler laws of stream numbers and lengths [8,9], although the geomorphological significance of these laws should be questioned [10–12]. These models assume simple boundary conditions at the outlet such as constant slope [7] or elevation [5]. Some also assume that the drainage is able to transport the product of their avalanchelike erosion formalism out of the system [13–15]. But the way in which boundary conditions influence the solution of this formalism is seldom discussed [6,16,17].

In typical drainage basins, the erosion products are transported either by tributaries or by slope processes to the main river or stream that carries these masses away from places of high relief to the sea where the grains ultimately accumulate in sedimentary basins (Fig. 1). When leaving the mountain range, the main stream forms a floodplain or alluvial plain, where sands and finer alluvium can temporarily accumulate. We focus here on the consequences on the floodplain behavior induced by a long-term constant output flux at a main stream outlet (sediment flux to the sea), whereas the upstream relief is quickly eroding; this is the case for the largest rivers of Asia for at least the past 2 million years [18,19]. Floodplain processes are modeled using a common first-order diffusive approximation resulting from simplification of the St. Venant-Exner equations [20–23]. Using a simple analytical formalism, we discuss in what way fluvial systems react to both changes of flux from hillslopes and from upstream catchments in order to maintain a constant boundary condition at the outlet [24].

II. DIFFUSIVELIKE RIVER BED EVOLUTION

Here we consider a very simple first-order (1+1)-dimensional [(1+1)D] model of a single floodplain reaction to an external forcing mechanism. The main stream is linear and has a floodplain (a plain where sands and alluvium accumulate) of length L (Fig. 1). The x axis is taken along the river bed, and its origin is located at the river mouth.

We start from a system of two equations known as the St. Venant-Exner equations. The equation of Exner defines the conservation of the solid mass transported by a river. In our (1+1)D approach it may be expressed as

$$(1-p)\partial_t z+\partial_u q_s \partial_x u=\phi(x,t), \tag{1}$$

where p is the sediment porosity, q_s is the sediment discharge, z is the river bed elevation above some reference datum, u is the flow velocity, and $\phi(x,t)$ is a source term characterizing mass input from adjacent slopes or tributaries.

The equation of St. Venant defines the dynamics of a gradually varied unsteady flow

$$g\partial_x(h+z)+\alpha u\partial_x u+\partial_t u=-gJ_e, \tag{2}$$

where h is the flow depth, α is the Coriolis or energy coefficient, and J_e is the energy slope of the flow [25,23]. The St. Venant equation expresses the conservation of energy principle. J_e represents the variation in the hydraulic head of the river due to turbulent and friction losses, $g\partial_x(h+z)$ is the variation in potential energy, $u\partial_x u$ is the variation of kinetic energy, and $\partial_t u$ is the time variations in momentum of the river. The basic assumption, attributed to St. Venant, who was among the first to derive this equation, is that the energy slope, or friction slope, of the flow is the same as that in a uniform flow and can therefore be estimated through the use of uniform flow formulas such as $J_e \sim u^2/C^2 h$, where C is the Chezy coefficient [23].

The simplifying assumptions we make are as follows: (i) we assume a quasistationary form of the St. Venant-Exner equations, which is a reasonable assumption for large rivers where the Froude number shows that the flow is clearly subcritical ($Fr=u/\sqrt{gh}<1$); (ii) we assume further that the flow is quasiuniform ($\partial_x u \sim 0$, $\partial_x h=0$) and quasipermanent ($\partial_t u \sim 0$). Equation (2) then reduces to

$$\partial_x z=-J_e=-u^2/C^2 h, \tag{3}$$

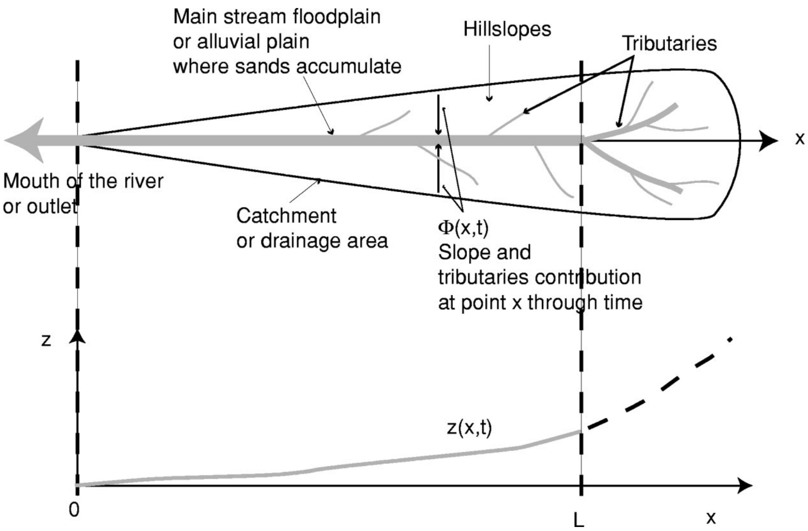

FIG. 1. Sketch of the model floodplain studied and terms used in the text.

where q is the unit flow discharge ($q=uh$) [20]. Differentiating this equation and substituting into Eq. (1) leads to

$$\nu\partial_{xx}^2 z=\partial_t z+\Phi(x,t) 0<x<L, 0<t<\infty, \tag{4}$$

where $\nu=\partial_u q_s C^2 h/[2u(1-p)]$ is a diffusivity coefficient and $\Phi(x,t)=\phi(x,t)/(1-p)$. The assumptions we make imply that (i) the simplified parabolic form of the river bed elevation is realistic for characteristic distances much larger than the flow depth, which is always true (we are looking for effects on scales of kilometers to hundreds of kilometers compared to flow depths of a few meters, hence $x\gg 3h/J_e$), (ii) the problem is to be considered only for very large time scales (which is also the case here, as climatic oscillations during the past 2 million years have had characteristic periods on order of 10^4 years), and finally (iii) we deal only with large streams where subcritical quasiuniform and quasipermanent flow is a reasonable approximation (the behavior we are trying to understand concerns rivers that have alluvial plains of length on the order of 1000 km). Thus we can reasonably assume, to the first order, a constant diffusivity $\nu(t)\approx Cte$ [22].

III. BOUNDARY CONDITIONS

Through a mass balanced reconstruction of the volumes accumulated at the outlet of large streams in Asia, our earlier work has shown that present-day measured fluxes of mass carried by large streams to the sea are equal to the average of these fluxes over a 2×10^6-year period [known as the quaternary, Fig. 2(a)] [18,19]. It can further be shown that these average fluxes are maximum with regard to the long-term history of these rivers (over periods larger than 10^7 years [Fig. 2(b)]). As we are interested in the study of rivers over the past 2 millions years, the first boundary condition, at the mouth of the river (in $x=0$), we assume is

$$\partial_x z(0,t)=\frac{\langle Q_f\rangle}{\nu w}, \quad 0<t<\infty, \tag{5}$$

where $\langle Q_f\rangle$ is the average sediment discharge at the outlet (L) of the catchment and w is the floodplain width at the outlet.

The second boundary condition (at the river source $x=L$) is

$$\alpha\partial_x z(L,t)+\beta z(L,t)=f(t), \quad 0<t<\infty, \tag{6}$$

where $\alpha([L])$ and β (dimensionless) are two constants and $f(t)([L])$ is a function of time. This Neuman-type boundary condition defines the very general relationship existing at the source of the river between the topography and its gradient. We will see later that, depending on the values of the parameters α, β, or the function $f(t)$, simple flux-type or elevation-type boundary conditions can be explored. The value of the function $f(t)$ physically depends on the conditions prevailing in the upper part of the river basin above the floodplain [22,26].

Eventually the initial conditions are taken as

$$z(x,0)=z_0(x), \tag{7}$$

where $z_0(x)$ is the initial topographic profile along the river bed.

IV. SOLUTION AND DISCUSSION

The floodplain model defined above has never been analytically studied. Previous analyses have focused on portions of a river assuming the river to be infinite or semi-infinite in length and leading to erfc-type behaviors [20]. Interesting numerical analyses have focused on the evolution of alluvial plains under different types of parametrization [21,22]. We therefore give and discuss the exact solution we obtain using a general eigenvalue solution of the problem.

A. Solution of the IBVP

The initial boundary value problem formed by Eqs. (4)–(7) can be solved analytically through separation of variables and eigenfunction expansion. Changing the variable $z(x,t)$ to $z(x,t)\rightarrow z(x,t)+[Q_f(\alpha+\beta L)-\nu w f(t)/\nu w L(2\alpha+\beta L)]x^2-(Q_f/\nu w)x$, the final solution of the problem is of the form

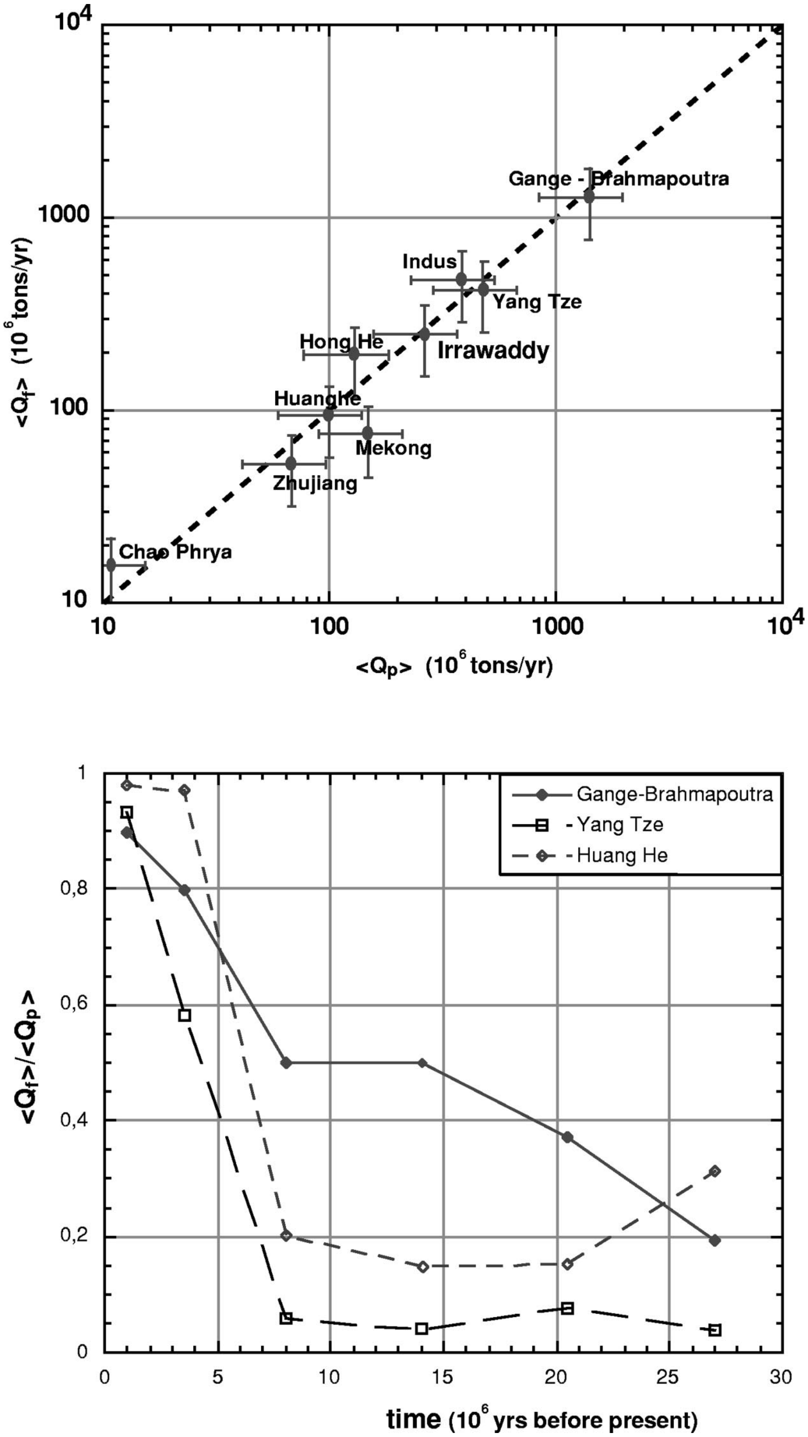

FIG. 2. (a) Present-day average mass fluxes ($\langle Q_p \rangle$) carried by large rivers of Asia at their outlet (drainage basins or catchments have characteristic size on the order of 10^5-10^6 km^2) compared to the total mass accumulated in the depressions at the outlets and averaged over 2×10^6 years ($\langle Q_f \rangle$). Each point represents one river. Note the remarkable one-to-one correlation between these two independant estimates. (b) Evolution of this mass flux ratio $\langle Q_f \rangle / \langle Q_p \rangle$ since approximately the last 25 million years for three different rivers of Asia, showing long-term convergence towards maximum present-day fluxes.

$$z(x,t)=\sum_{n=1}^{n=\infty} a_n e^{-(\sqrt{\nu}\lambda_n/L)^2 t}\cos\frac{\lambda_n x}{L} + \sum_{n=1}^{n=\infty}\cos\frac{\lambda_n x}{L}\left[\int_0^t e^{-(\sqrt{\nu}\lambda_n/L)^2(t-\tau)}\Psi_n(\tau)\,d\tau\right] - \frac{Q_f(\alpha+\beta L)-\nu w f(t)}{\nu w L(2\alpha+\beta L)}x^2+\frac{Q_f}{\nu w}x, \tag{8}$$

where the eigenvalues λ_n are the solutions of (Fig. 3)

$$\tan(\lambda_n)=\frac{\beta L}{\alpha\lambda_n} \tag{9}$$

and the coefficients a_n and $\Psi_n(t)$ are of the form

$$a_n=\frac{2\lambda_n}{L(\sin 2\lambda_n+2\lambda_n)}\times\int_0^L\left(z_0(x)+\frac{Q_f(\alpha+\beta L)-\nu w f(t)}{\nu w L(2\alpha+\beta L)}x^2-\frac{Q_f}{\nu w}x\right)\times\cos\left(\frac{\lambda_n x}{L}\right)dx\,, \tag{10}$$

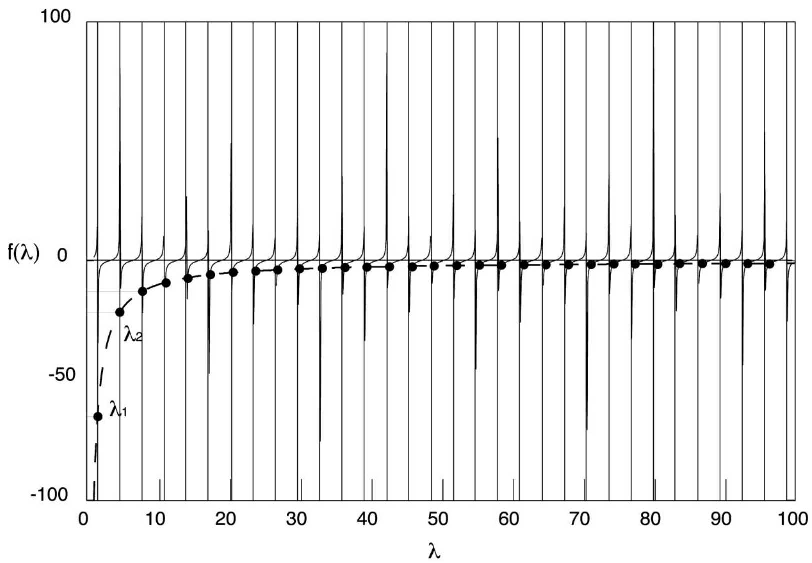

FIG. 3. Graphical representation of the eigenvalue problem defined by Eq. (9) for a steady-state concave profile of the river bed; $\alpha=1$, $\beta=-1$, $L=100$. Solid line: $f(\lambda)=\tan(\lambda)$; dashed line: $f(\lambda)=\beta L/\alpha\lambda$; dots are solutions of the eigenvalue problem: $\tan(\lambda)=\beta L/\alpha\lambda$.

$$\Psi_n(t)=\frac{2\lambda_n}{L(\sin 2\lambda_n+2\lambda_n)}\int_0^L\left(\Phi(x,t)-\frac{2Q_f(\alpha+\beta L)}{wL(2\alpha+\beta L)}+\frac{2\nu f(t)-\partial_t f(t)x^2}{L(2\alpha+\beta L)}\right)\cos\left(\frac{\lambda_n x}{L}\right)dx. \quad (11)$$

B. Asymptotic profile of a river floodplain

The predictive nature of Eq. (8) reveals some information concerning the way changes in hillslope erosion can induce reaction of the floodplain. For large time spans, the solution asymptotically tends towards

$$z(x,t)=\frac{Q_f}{\nu w}x-\frac{Q_f(\alpha+\beta L)-\nu w f(t)}{\nu w L(2\alpha+\beta L)}x^2+\sum_{n=1}^{n=\infty}\cos\frac{\lambda_n x}{L}\left[\int_0^t e^{-(\lambda_n\sqrt{\nu}/L)^2(t-\tau)}\Psi_n(\tau)d\tau\right]. \quad (12)$$

Equation (12) has two components. The second-order polynomial function reflects the importance of the boundary conditions for the average profile of a floodplain. It also reflects the evolution of a river system that will tend to achieve a profile in which the elevation at the outlet is at a constant level (in this case sea level). The cosine series represents the induced perturbations of this profile through a direct contribution from hillslopes and tributaries to the floodplain.

Depending on the values of α, β and the form of $f(t)$ that define the boundary condition at the upper reach of the floodplain, the average profile will be concave, linear, or convex. Given the fact that most present-day, long-distance, alluvial river profiles are linear or very slightly concave upwards (average gradients for a large river are on the order of 10^{-3-4}), we get $f(t)\geqslant Q_f(\alpha+\beta L)/\nu w$.

In the case $\alpha=0,\beta\neq 0$, we get $z(L)=f(t)/\beta\geqslant Q_f L/\nu w$ (and the eigenvalues λ_n are of the form $\lambda_n=n\pi/2$), which means that the elevation of the floodplain scales its length and with the output flux.

In the case $\alpha\neq 0,\beta=0$, we get $\partial_x z(L)=f(t)/\alpha\geqslant Q_f/\nu w$ (and the eigenvalues λ_n are of the form $\lambda_n=n\pi$). Concavity of the floodplain implies that the input flux from the slopes is larger than the output flux at the mouth of the river.

In both cases, upward concavity of the profiles reflects the process by which the river actively stores sediments in its floodplain.

If one assumes a linear steady profile of the river plain, then it is possible to estimate the diffusivity coefficient provided one has the knowledge of $(z(L),L,Q_f,w)$ in the first case and $(\partial_x z(L),Q_f,w)$ in the second case. Using the parameters derived by Humphrey and Heller [22] in a numerical model of a steady evolving floodplain [$z(L)=200$ m, $L=10^5$ m, $\partial_x z\sim 0{,}02$, $w=10^3$ m, $Q_f=5000$ m^3 yr^{-1}], we find in both case the same diffusivity $\nu=250$ m^2 yr^{-1} equal to the one they derive.

C. Hillslope perturbations and reaction of the river

The second term in Eq. (12), the hillslope and tributaries mass flux term (this flux is modeled in very different ways in the literature, i.e., avalanches, diffusion, or kinematic wave propagation), expresses the time dependence of the river bed elevation. This control is exerted through the integral term $\int_0^t e^{-(\lambda_n\sqrt{\nu}/L)^2(t-\tau)}\Psi_n(\tau)d\tau$, which is dependant on the form of $\Phi(x,t)$ and $f(x,t)$ [see Eq. (11)], hence on time variations of hillslope transport to the floodplain.

Thus a river will aggrade (deposit its load) or degrade (erode and transport grains) in order to buffer hillslope flux cycles and keep a constant mass flux (boundary conditions) at the mouth of the river. This hillslope and tributaries flux term, which may be controlled either by climate, tectonics (uplift), or internal instabilities, thus acts as a perturbation against the establishment of a linear steady-state equilibrium profile of the river.

For example, assuming $f(t)=A_0(1+\sin\omega t)$ and $\Phi(x,t)=(\Phi_0/L)x\sin(\xi t)$, two very simple oscillatory functions that stand for the supply from the upper end part of the catchment and from the side slopes of the river plain through time, respectively, Eqs. (11) and (12) lead to an asymptotic stationary solution of the form

$$z(x,t)=\frac{Q_f}{\nu w}x-\frac{Q_f(\alpha+\beta L)-\nu w A_0(1+\sin\omega t)}{\nu w L(2\alpha+\beta L)}x^2+\sum_{n=1}^{n=\infty}\cos\left(\frac{\lambda_n x}{L}\right)\left(\frac{1}{\lambda_n\eta_n}\left\{b_n+\frac{1}{1+\left(\frac{\omega}{\eta_n}\right)^2}\times\left[c_n\left(\sin\omega t-\frac{\omega}{\eta_n}\cos\omega t\right)-d_n\left(\cos\omega t+\frac{\omega}{\eta_n}\sin\omega t\right)\right]+e_n\frac{1}{1+\left(\frac{\xi}{\eta_n}\right)^2}\left(\sin\xi t-\frac{\xi}{\eta_n}\cos\xi t\right)\right\}\right), \tag{13}$$

where $\eta_n=\lambda_n^2\nu/L^2$ are the characteristic reaction frequencies of the river plain associated to the eigenvalues, and λ_n, b_n, c_n, d_n, and e_n are constant values for each n and are such that $\lim_{n\to\infty}(b_n/\lambda_n),(c_n/\lambda_n),(d_n/\lambda_n),(e_n/\lambda_n)=0$.

From Eq. (13), two interesting cases arise.

(a) The frequency of the perturbation is larger than some characteristic frequency of the river system: $\exists k\in\mathrm{IN}/(\omega/\eta_n)\gg 1, n\leqslant k(\xi/\eta_n\gg 1)$. In this case the solution reduces to

$$z(x,t)=\frac{Q_f}{\nu w}x-\frac{Q_f(\alpha+\beta L)-\nu w A_0(1+\sin\omega t)}{\nu w L(2\alpha+\beta L)}x^2+\sum_{n=1}^{n=\infty}\frac{b_n}{\lambda_n\eta_n}\cos\left(\frac{\lambda_n x}{L}\right)+O(k), \tag{14}$$

where

$$O(k)=\sum_{n=k}^{n=\infty}\frac{1}{\lambda_n\eta_n}\cos\left(\frac{\lambda_n x}{L}\right)\{c_n\sin\omega t-d_n\cos\omega t+e_n\sin\xi t\} \tag{15}$$

represents very-small-amplitude perturbations as $\lambda_n\eta_n$ rapidly tends towards infinite values. The temporal oscillations of slope, tributaries, and upper catchment inputs are buffered (apart from the first-order oscillation of the second-order polynom).

(b) The frequency of the perturbation is always smaller than the characteristic frequencies of the river system: $\forall n\in\mathrm{IN}, \omega/\eta_n\ll 1(\xi/\eta_n\ll 1)$. In this case the solution tends towards

$$z(x,t)=\frac{Q_f}{\nu w}x-\frac{Q_f(\alpha+\beta L)-\nu w A_0(1+\sin\omega t)}{\nu w L(2\alpha+\beta L)}x^2+\sum_{n=1}^{n=\infty}\frac{1}{\lambda_n\eta_n}\cos\left(\frac{\lambda_n x}{L}\right)\{b_n+c_n\sin\omega t-d_n\cos\omega t+e_n\sin\xi t\}. \tag{16}$$

Temporal oscillations of slope, tributaries, and upper catchment inputs are not buffered and the alluvial plain is going to react to perturbations by cyclic aggradation (deposition) and degradation (erosion) waves along the plain.

Thus for perturbations from the slopes to induce a complex reaction of the alluvial plain, the frequency of the pertubation has to be much less than those of the river bed. Very seldom is this the case in reality, where the last known climatic perturbations have frequencies on the order of 10^{-4} yr^{-1} [28], whereas large Asian river plains have minimum frequencies or the order of 10^{-5-6} yr^{-1} [27,19].

D. River saturation

One question that remains in the end is why should the river maintain such a constant flux at the outlet? The simplest answer is that the average transport capacity of a river may reach some long-term maximum value (hence $\partial^2_{tx}z\sim 0$), as suggested by Fig. 2(b). Fluctuations may be observed at places through time but the average transport capacity along the entire river bed is approximately constant, and maximum. Returning to Eq. (8), we can see that the time derivative of the mass flux ($\partial_x z$) is going to be of the form

$$\partial^2_{tx}z(x,t)=\frac{2x}{L(2\alpha+\beta L)}\partial_t f(t)-\sum_{n=1}^{n=\infty}\frac{\lambda_n}{L}\sin\left(\frac{\lambda_n x}{L}\right)\partial_t F_n(t), \tag{17}$$

where

$$F_n(t)=\int_0^t e^{-(\sqrt{\nu}\lambda_n/L)^2(t-\tau)}\Psi_n(\tau)d\tau. \tag{18}$$

Equation (17) is made up of two terms that will tend to cancel each other. The first term on the right-hand side corresponds to the temporal variations of the mass input at the upper end of the floodplain. The second term corresponds to the reaction induced by these changes in fluxes.

For $\partial^2_{tx}(x,t)$ to be approximately equal to zero, one needs to have either these two terms canceling each other or the condition that these two terms are negligible, hence $\partial_t f(t)\sim 0$ and $\partial_t\Phi(x,t)\sim 0$. In the first case, the river reacts very rapidly to changes in input fluxes, whereas in the second one, slope erosion and mass coming in from the upstream end of the floodplain are basically constant. This latter condition is possible for rivers that have a catchment in high mountains such as the Himalayas. In these regions it has been shown that rapid active uplift can lead to a strong control on river incision [29]. Rivers in the upper catchment may therefore be forced to incise at a constant rapid rate despite climate oscillations. The Indus river, for example, has been carving deep gorges at about 1 cm/yr in its upper course (before it forms a large alluvial plain), in order to keep pace with the rapid uplift of the Nanga Parbat Haramosh massifs (western Himalayas) [29]. In these regions, erosion in the catchment is so high that changes in climate do not seem, on average, to affect erosion fluxes [hence $\partial_t f(t)\sim 0$ and $\partial_t\Phi(x,t)\sim 0$]. In that case, the commonly used assumption of constant rate lowering of the base level, which mimics constant and uniform erosion rates, may then be taken as a reasonable boundary condition for models trying to catch stationary forms of landscape evolution [3,6].

V. SUMMARY

Under the first-order linear diffusion approximation, simple analytical solutions can be derived. Assuming a river of finite length and realistic boundary conditions especially the boundary specifying the flux of mass carried by a river at its mouth can lead to simple analytic solutions carrying a wealth of information. Indeed, it seems that present-day flux of large Asian rivers at their mouth remains on average constant. This can be explained by diffusive buffering of the floodplain reacting to changes in the erosion on hillslopes and fluxes from the tributaries. We suggest that the characteristic frequencies of the river system control the frequencies of the perturbation that can pass through and induce oscillations of the river plain. The fact that, at present, erosion fluxes leaving mountainous areas seem to be maximum with regard to the present size of the river systems [$\sim 10^6$ km^2, Fig. 2(b)] suggests that the reason for the saturation of the river may rely on the rapid uplift of eroding mountain ranges. This uplift may exert strong control over erosion rates, an influence that is common to most landscape evolution models through the use of a boundary condition such as $\partial_t z(r)_{r=0}=$ constant.

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[1] G. Wilgoose, R. Bras, and I. Rodriguez-Iturbe, Water Resour. Res. **27**, 1671 (1991).
[2] A. Czirók, E. Somfai, and T. Vicsek, Phys. Rev. Lett. **71**, 2154 (1993).
[3] A. Howard, W. Dietrich, and M. Seidl, J. Geophys. Res. **99**, 13 971 (1994).
[4] L. Gil and D. Sornette, Phys. Rev. Lett. **76**, 3991 (1996).
[5] K. Sinclair and R. Ball, Phys. Rev. Lett. **76**, 3360 (1996).
[6] J. Banavar *et al.*, Phys. Rev. Lett. **78**, 4522 (1997).
[7] E. Somfai and L. Sander, Phys. Rev. E **56**, R5 (1997).
[8] R. Horton, Geol. Soc. Am. Bull. **56**, 375 (1945).
[9] I. Rodriguez-Iturbe and A. Rinaldo, *Fractal River Basins* (Cambridge University Press, Cambridge, 1997), p. 547.
[10] J. Kirchner, Geology **21**, 591 (1993).
[11] J. Kirchner, Geology **22**, 380 (1994).
[12] J. Masek and D. Turcotte, Geology **22**, 380 (1994).
[13] H. Takayasu and H. Inaoka, Phys. Rev. Lett. **68**, 966 (1992).
[14] A. Rinaldo *et al.*, Phys. Rev. Lett. **70**, 822 (1993).
[15] R. Leheny, Phys. Rev. E **52**, 5610 (1995).
[16] A. Howard, Earth Surf. Processes **22**, 211 (1997).
[17] The study of Banavar is one of the rare examples that considers the issue of boundary conditions. However, a literal interpretation of their equation at the upstream boundary (without the addition of a smoothing diffusive term) would lead to an unrealistic infinite gradient for the main streams at their source [$\partial_x z(0,t) \rightarrow \infty$]. This, however, was not the primary focus of their study.
[18] F. Métivier, Ph.D. thesis, Université Paris 7, 1996.
[19] F. Métivier and Y. Gaudemer, Basin Res. (to be published).
[20] J. Soni, Water Resour. Res. **17**, 33 (1981).
[21] C. Paola, P. Heller, and C. Angevine, Basin Res. **7**, 73 (1992).
[22] N. Humphrey and P. Heller, Geology **23**, 499 (1995).
[23] W. Graf and M. Altinakar, *Hydraulique Fluviale, Ecoulement Nonpermanent et Phénomènes de Transport* (Eyrolles, Paris, 1996), p. 378.
[24] We shall speak indifferently of river, river system, river plain, or alluvial plain, keeping in mind that the assumptions we make and analyses we derive are only valid for large alluvial rivers.
[25] V. T. Chow, *Open-channel Hydraulics* (McGraw Hill, Singapore, 1959).
[26] Humphrey and Heller, in a numerical analysis of a similar problem, assume that $f(t)$ varies through time as the result of a kinematic wave propagation of the erosion products from the slopes to the river.
[27] W. Dade and P. Friend, J. Geol. **106**, 661 (1998).
[28] J. Chappel and N. Shackelton, Nature (London) **324**, 137 (1986).
[29] D. Burbank *et al.*, Nature (London) **379**, 505 (1996).